Exercice 7.

Mettre sous forme exponentielle les nombres complexes suivants :

$z_1 = 3 + 3i \qquad z_2 = -1 - \sqrt{3}i \qquad z_3 = \frac{-4}{3}i \qquad z_4 = -2$

$|z_1| = |3 + 3i| = \sqrt{9 + 9} = 3\sqrt{2}$

$$\left.\begin{array}{l} \cos\theta_1 = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\theta_1 = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{array}\right\} \text{donc } \theta_1 = \frac{\pi}{4}\ [2\pi]$$

donc $z_1 = \mathbf{3\sqrt{2}\, e^{i\frac{\pi}{4}}}$

$|z_2| = \left|-1 - \sqrt{3}i\right| = \sqrt{1 + 3} = \sqrt{4} = 2$

$$\left.\begin{array}{l} \cos\theta_2 = -\frac{1}{2} \\ \sin\theta_2 = \frac{-\sqrt{3}}{2} \end{array}\right\} \text{donc } \theta_2 = \frac{-2\pi}{3}\ [2\pi]$$

donc $z_2 = \mathbf{2\, e^{\frac{-2i\pi}{3}}}$

$z_3 = \frac{-4}{3}i = \mathbf{\frac{4}{3}e^{\frac{-i\pi}{2}}}$

$z_4 = -2 = \mathbf{2e^{-i\pi}}$

Exercice 8.

Mettre sous forme exponentielle les nombres complexes suivants :

$z_1 = \frac{\sqrt{2}}{1 - i} \qquad z_2 = \sqrt{3} + 3i \qquad z_3 = {z_1}^2 \qquad z_4 = -\sqrt{6} - i\sqrt{2}$

$$z_1 = \frac{\sqrt{2}}{1 - i} = \frac{\sqrt{2}(1 + i)}{(1 - i)(1 + i)} = \frac{\sqrt{2} + i\sqrt{2}}{1 - i^2} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

$$|z_1| = \left|\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right| = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{\frac{4}{4}} = 1$$

$$\left.\begin{array}{l} \cos\theta_1 = \dfrac{\sqrt{2}}{2} \\ \sin\theta_1 = \dfrac{\sqrt{2}}{2} \end{array}\right\} \text{donc } \theta_1 = \frac{\pi}{4}\ [2\pi]$$

donc $z_1 = \boldsymbol{e^{i\frac{\pi}{4}}}$

$$|z_2| = |\sqrt{3} + 3i| = \sqrt{3+9} = \sqrt{12} = \sqrt{4\times 3} = \sqrt{4}\times\sqrt{3} = 2\sqrt{3}$$

$$\left.\begin{array}{r} \cos\theta_2 = \dfrac{\sqrt{3}}{2\sqrt{3}} = \dfrac{1}{2} \\ \sin\theta_2 = \dfrac{3}{2\sqrt{3}} = \dfrac{3\times\sqrt{3}}{2\sqrt{3}\times\sqrt{3}} = \dfrac{3\sqrt{3}}{2\times 3} = \dfrac{\sqrt{3}}{2} \end{array}\right\} \text{donc } \theta_2 = \frac{\pi}{3}\ [2\pi]$$

donc $z_2 = \boldsymbol{2\sqrt{3}\, e^{\frac{i\pi}{3}}}$

$$z_3 = {z_1}^2 = \left(\frac{\sqrt{2}}{1-i}\right)^2 = \frac{2}{(1-i)^2} = \frac{2}{1-2i+i^2} = \frac{2}{1-2i-1} = \frac{2}{-2i} = \frac{1\times i}{-i^2} = i$$

$z_3 = i = \boldsymbol{1\times e^{\frac{i\pi}{2}}}$

$$|z_4| = |-\sqrt{6} - i\sqrt{2}| = \sqrt{6+2} = \sqrt{8} = \sqrt{4\times 2} = \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$$

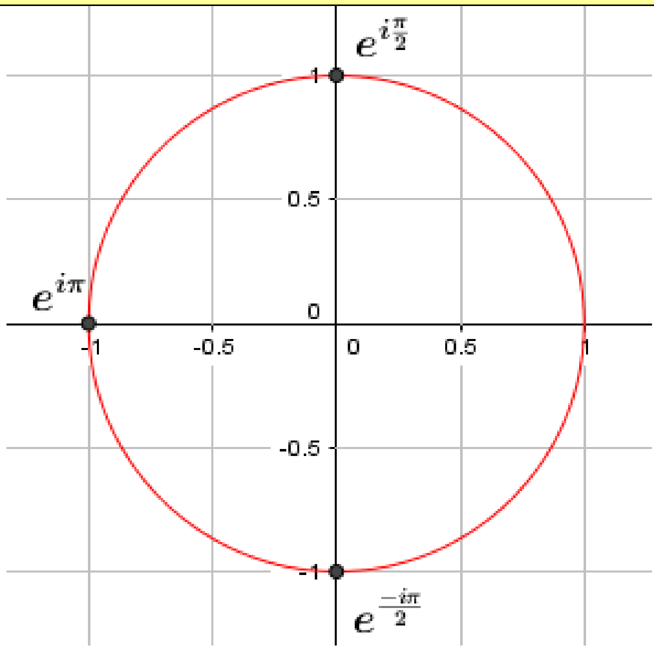
$$\left.\begin{array}{r} \cos\theta_4 = \dfrac{-\sqrt{6}}{2\sqrt{2}} = \dfrac{-\sqrt{3}\times\sqrt{2}}{2\times\sqrt{2}} = \dfrac{-\sqrt{3}}{2} \\ \sin\theta_4 = \dfrac{-\sqrt{2}}{2\sqrt{2}} = \dfrac{-1}{2} \end{array}\right\} \text{donc } \theta_4 = \frac{-5\pi}{6}\ [2\pi]$$

donc $z_4 = \boldsymbol{2\sqrt{2}\, e^{\frac{-5i\pi}{6}}}$

Exercice 9.

▶1. Dans le plan muni d'un repère orthonormal $(O,\vec{u},\vec{v})$, placer les points d'affixes

$$e^{i\pi} \qquad e^{i\frac{\pi}{2}} \qquad e^{-i\frac{\pi}{2}}$$



▶ 2. Soit $k = \dfrac{1}{e^{i\pi}} - e^{2i\pi} + \dfrac{e^{\frac{i\pi}{2}}}{e^{-\frac{i\pi}{2}}}$. Est-il vrai ou faux que k est un nombre réel ?

$e^{i\pi} = -1$
$e^{\frac{i\pi}{2}} = i$
$e^{-\frac{i\pi}{2}} = -i$
$e^{2i\pi} = 1$

donc $k = \dfrac{1}{e^{i\pi}} - e^{2i\pi} + \dfrac{e^{\frac{i\pi}{2}}}{e^{-\frac{i\pi}{2}}} = \dfrac{1}{-1} - 1 + \dfrac{i}{-i} = -1 - 1 - 1 = -3$

k est donc un nombre réel.