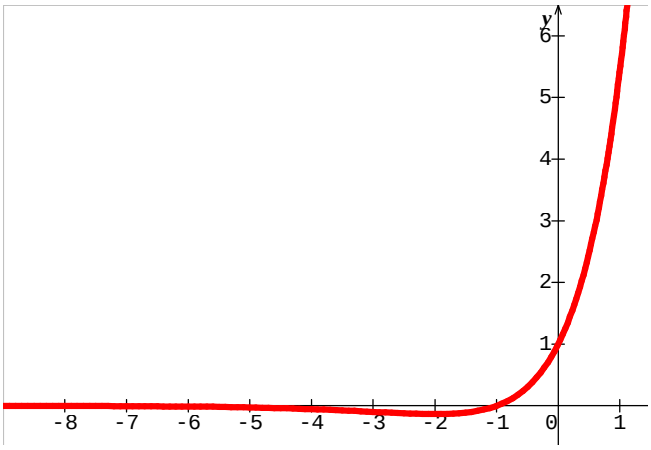
SUJET A

Exercice 1

1a

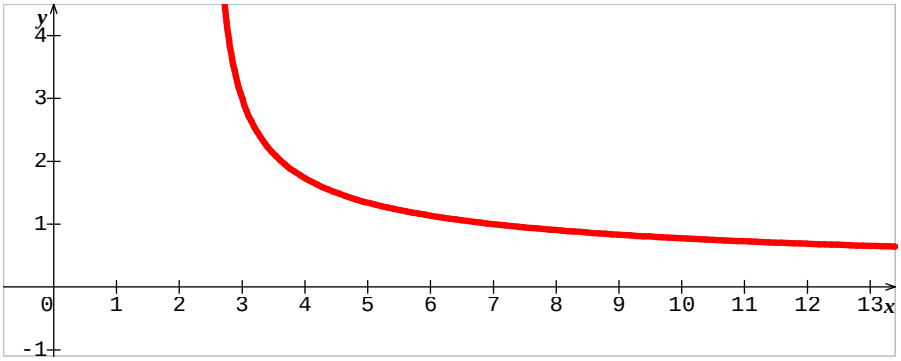
$\left.\begin{array}{l}\lim\limits_{x\to+\infty} x+1=+\infty \\ \lim\limits_{x\to+\infty} e^x=+\infty\end{array}\right\}$ par produit $\lim\limits_{x\to+\infty}(x+1)e^x=+\infty$



1b

$\lim\limits_{x\to+\infty} 2x-5=+\infty$ donc $\lim\limits_{x\to+\infty}\sqrt{2x-5}=+\infty$

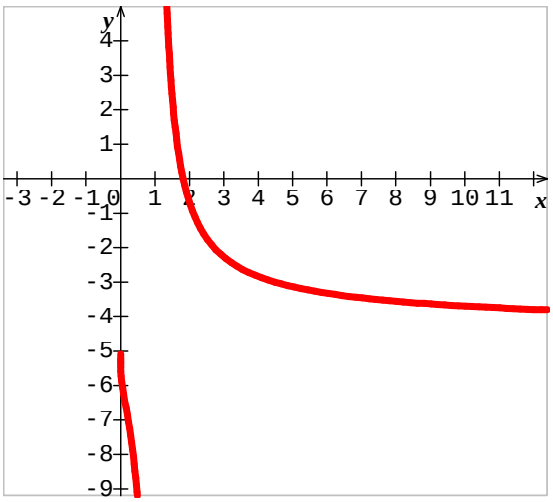
et donc $\lim\limits_{x\to+\infty}\dfrac{3}{\sqrt{2x-5}}=0$



1c

$\lim\limits_{x\to0}\ln x=-\infty$ donc $\lim\limits_{x\to0}\dfrac{3}{\ln x}=0$

$\left.\begin{array}{l}\lim\limits_{x\to0} x-5=-5 \\ \lim\limits_{x\to0}\dfrac{3}{\ln x}=0\end{array}\right\}$ par addition $\lim\limits_{x\to0} x-5+\dfrac{3}{\ln x}=-5$



Exercice 1

2a

$\left.\begin{array}{l}\lim\limits_{x\to+\infty} 1-5x=-\infty \\ \lim\limits_{x\to+\infty} x-3=+\infty\end{array}\right\}$ par quotient la forme est indéterminée

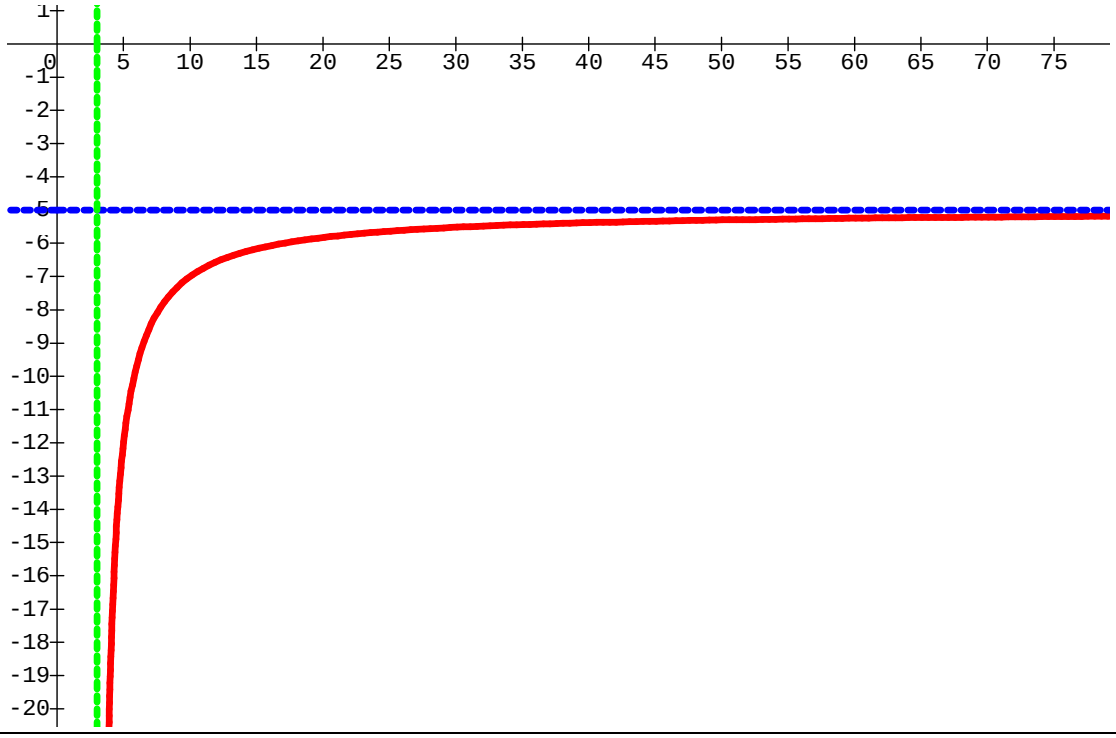
or $\lim\limits_{x\to+\infty} \dfrac{1-5x}{x-3} = \lim\limits_{x\to+\infty} \dfrac{-5x}{x} = -5$

La courbe de f admet donc une asymptote horizontale : $y=-5$.

2b

$\left.\begin{array}{l}\lim\limits_{x\to3^+} 1-5x=-14 \\ \lim\limits_{x\to3^+} x-3=0^+\end{array}\right\}$ donc $\lim\limits_{x\to3^+} f(x)=-\infty$

La courbe de f admet donc une asymptote verticale : $x=3$.



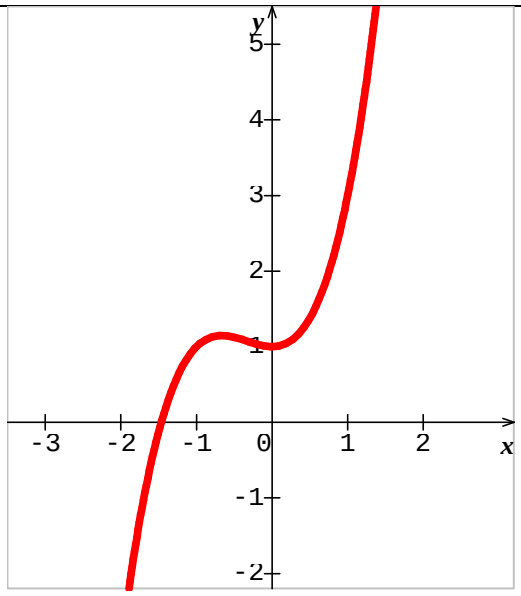
3a

$\left.\begin{array}{l}\lim\limits_{x\to+\infty} x^3=+\infty \\ \lim\limits_{x\to+\infty} x^2+1=+\infty\end{array}\right\}$ par somme $\lim\limits_{x\to+\infty} x^3+x^2+1=+\infty$

3b

$\left.\begin{array}{l}\lim\limits_{x\to-\infty} x^3=-\infty \\ \lim\limits_{x\to-\infty} x^2+1=+\infty\end{array}\right\}$ par somme la forme est indéterminée

or $\lim\limits_{x\to-\infty} x^3+x^2+1 = \lim\limits_{x\to-\infty} x^3 = -\infty$



Exercice 2.

1.

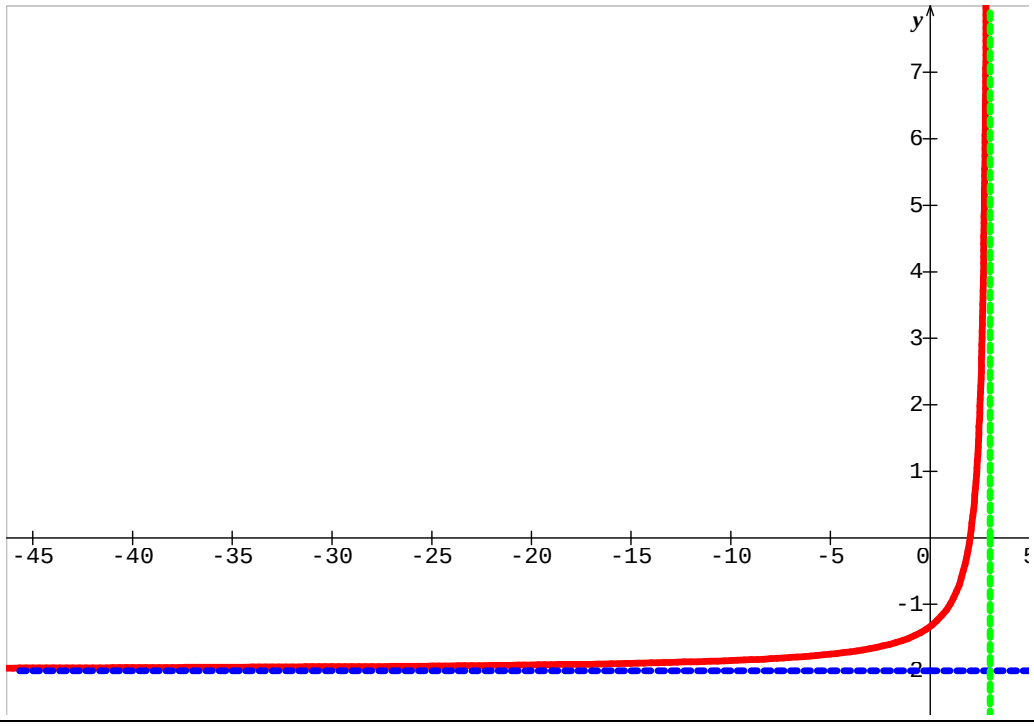
$\left.\begin{array}{l}\lim\limits_{x\to-\infty} 2x-4=-\infty \\ \lim\limits_{x\to-\infty} 3-x=+\infty\end{array}\right\}$ par quotient la forme est indéterminée

or $\lim\limits_{x\to-\infty} \dfrac{2x-4}{3-x} = \lim\limits_{x\to-\infty} \dfrac{2x}{-x} = -2$

La courbe de f admet donc une asymptote horizontale : $y=-2$.

$\left.\begin{array}{l}\lim\limits_{x\to3^-} 2x-4=2 \\ \lim\limits_{x\to3^-} 3-x=0^-\end{array}\right\}$ donc $\lim\limits_{x\to3^-} f(x)=-\infty$

La courbe de f admet donc une asymptote verticale : $x=3$.



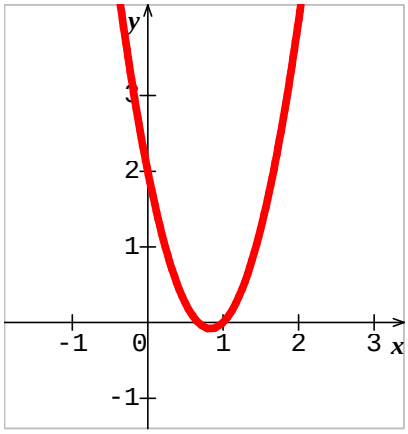
Exercice 3.

1.

$\left.\begin{array}{l}\lim\limits_{x\to+\infty} 3x^2 = +\infty \\ \lim\limits_{x\to+\infty} -5x+2 = -\infty\end{array}\right\}$ par addition la forme est indéterminée

or $\lim\limits_{x\to+\infty} 3x^2 - 5x + 2 = \lim\limits_{x\to+\infty} 3x^2 = +\infty$

$\left.\begin{array}{l}\lim\limits_{x\to-\infty} 3x^2 = +\infty \\ \lim\limits_{x\to-\infty} -5x+2 = +\infty\end{array}\right\}$ par addition $\lim\limits_{x\to-\infty} 3x^2 - 5x + 2 = +\infty$



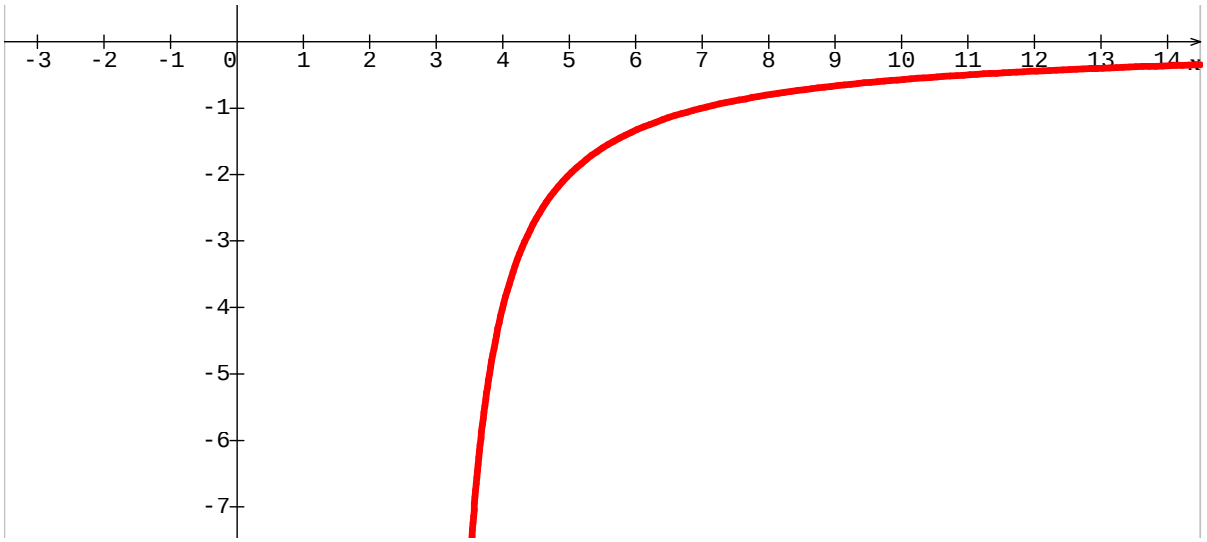
2.

$\lim\limits_{x\to+\infty} 3 - x = -\infty$ donc $\lim\limits_{x\to+\infty} \dfrac{4}{3-x} = 0$

La courbe de f admet donc une asymptote horizontale : $y = 0$.

$\left.\begin{array}{l}\lim\limits_{x\to3^+} 4 = 4 \\ \lim\limits_{x\to3^+} 3 - x = 0^-\end{array}\right\}$ donc $\lim\limits_{x\to3^+} g(x) = -\infty$

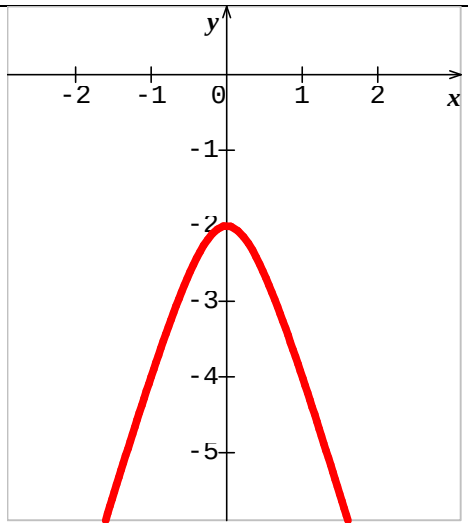
La courbe de f admet donc une asymptote verticale : $x = 3$.



3.

$\lim\limits_{x\to+\infty} 3x^2 + 1 = +\infty$ donc $\lim\limits_{x\to+\infty} h(x) = \lim\limits_{x\to+\infty} -2\sqrt{3x^2+1} = -\infty$

$\lim\limits_{x\to-\infty} 3x^2 + 1 = +\infty$ donc $\lim\limits_{x\to-\infty} h(x) = \lim\limits_{x\to-\infty} -2\sqrt{3x^2+1} = -\infty$



Exercice 3.

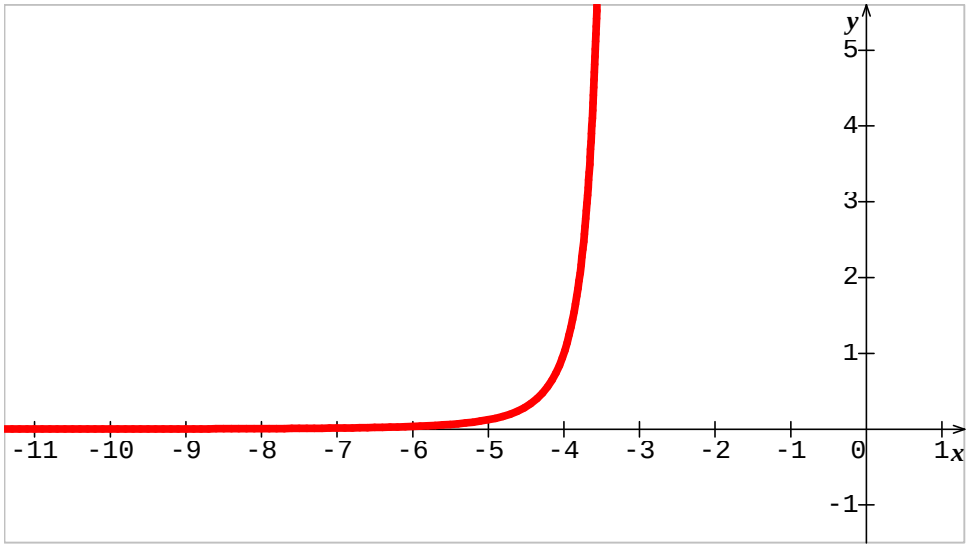
4.

$\lim\limits_{x\to-\infty} x+3=-\infty$ donc $\lim\limits_{x\to-\infty} \dfrac{-1}{(x+3)^3}=0$

La courbe de f admet donc une asymptote horizontale : $y=0$.

$\lim\limits_{x\to-3^-} x+3=0^-$ donc $\lim\limits_{x\to-3^-} \dfrac{-1}{(x+3)^3}=+\infty$

La courbe de f admet donc une asymptote verticale : $x=-3$.

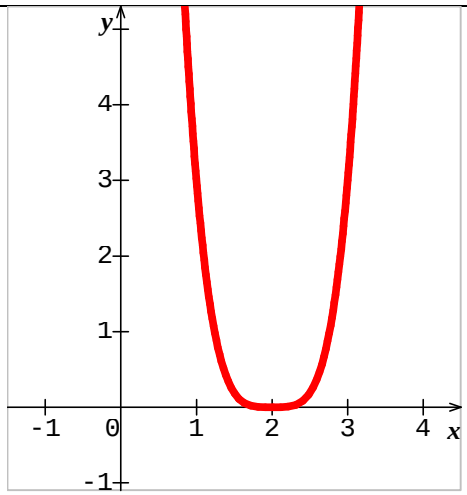


Exercice 4.

1.

$\lim\limits_{x\to-\infty} x-2=-\infty$ donc $\lim\limits_{x\to-\infty} 3(x-2)^4=+\infty$

$\lim\limits_{x\to+\infty} x-2=+\infty$ donc $\lim\limits_{x\to+\infty} 3(x-2)^4=+\infty$



Exercice 4.

2.

$\lim\limits_{x\to-\infty} 3x^2+1 = +\infty$ donc $\lim\limits_{x\to-\infty} \ln(3x^2+1) = +\infty$

$\lim\limits_{x\to+\infty} 3x^2+1 = +\infty$ donc $\lim\limits_{x\to+\infty} \ln(3x^2+1) = +\infty$