Question Flash n°3

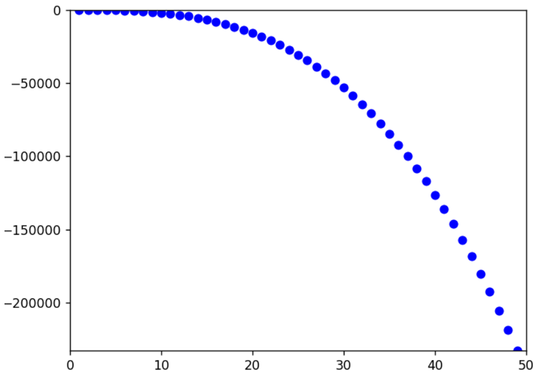
Déterminer les limites des suites définies ci-dessous :

$$a)\ \forall n \in \mathbb{N}, \qquad u_n = n^2 - 2n^3 - 7$$

$$b)\ \forall n \in \mathbb{N}, \qquad v_n = \sqrt{n^2+1} - n$$

$$c)\ \forall n \in \mathbb{N}, \qquad w_n = 3 \times (-1)^n - 2n$$

$$d)\ \forall n \in \mathbb{N}, \qquad y_n = \frac{3-n}{n^2+1}$$



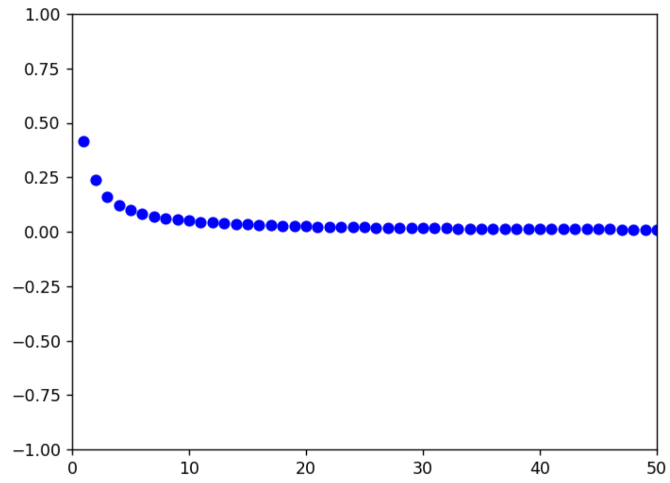
$$\boldsymbol{a)\ \forall n \in \mathbb{N}, \qquad u_n = n^2 - 2n^3 - 7}$$

$$\left.\begin{aligned}\lim_{n\to+\infty} n^2 = +\infty \\ \lim_{n\to+\infty} -2n^3 - 7 = -\infty\end{aligned}\right\}\text{FI par somme}$$

$$\forall n \in \mathbb{N}^*, \qquad u_n = n^3\left(\frac{1}{n} - 2 - \frac{7}{n^3}\right)$$

$$\left.\begin{aligned}\lim_{n\to+\infty} n^3 = +\infty \\ \lim_{n\to+\infty} \frac{1}{n} - 2 - \frac{7}{n^3} = -2\end{aligned}\right\}\text{donc, par produit, } \boldsymbol{\lim_{n\to+\infty} u_n = -\infty}$$

$b)\ \forall n \in \mathbb{N}, \qquad v_n = \sqrt{n^2+1} - n$



$$\left.\begin{array}{l}\lim\limits_{n\to+\infty} \sqrt{n^2+1} = +\infty \\ \lim\limits_{n\to+\infty} -n = -\infty\end{array}\right\}\text{FI par somme}$$

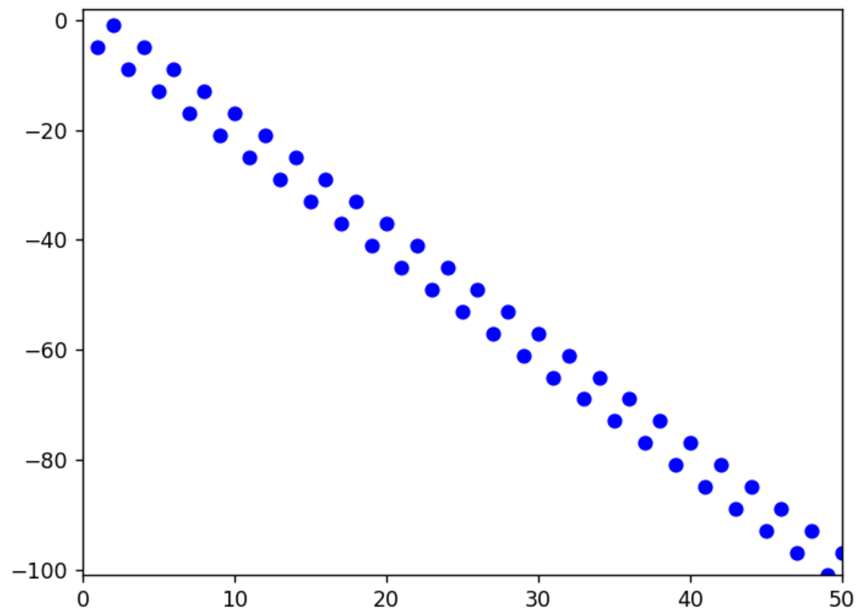
$$\forall n \in \mathbb{N}, v_n = \frac{\left(\sqrt{n^2+1}-n\right)\left(\sqrt{n^2+1}+n\right)}{\sqrt{n^2+1}+n}$$

$$v_n = \frac{\sqrt{n^2+1}^2 - n^2}{\sqrt{n^2+1}+n} = \frac{n^2+1-n^2}{\sqrt{n^2+1}+n} = \frac{1}{\sqrt{n^2+1}+n}$$

$\lim\limits_{n\to+\infty} \sqrt{n^2+1}+n = +\infty$ donc, par quotient, $\boldsymbol{\lim\limits_{n\to+\infty} v_n = 0}$

$c)$ $\forall n \in \mathbb{N}$,

$$\boldsymbol{w_n = 3 \times (-1)^n - 2n}$$



Je sais que, $\forall n \in \mathbb{N}, \quad -1 \leq (-1)^n \leq 1$

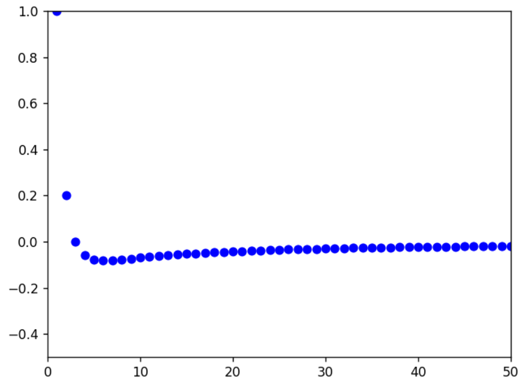
$$\Leftrightarrow -3 \leq 3 \times (-1)^n \leq 3$$

$$\Leftrightarrow -3 - 2n \leq 3 \times (-1)^n \leq 3 - 2n$$

$$\text{or, } \lim_{n \to +\infty} 3 - 2n = -\infty$$

donc, par comparaison, $\boldsymbol{\lim_{n \to +\infty} w_n = -\infty}$

$d)$ $\forall n \in \mathbb{N}$, $\quad y_n = \dfrac{3-n}{n^2+1}$



$$\left.\begin{array}{l}\lim\limits_{n\to+\infty} 3-n=-\infty \\ \lim\limits_{n\to+\infty} n^2+1=+\infty\end{array}\right\}\text{FI par quotient}$$

$$\forall n \in \mathbb{N}^*, \qquad y_n = \frac{n\left(\frac{3}{n}-1\right)}{n^2\left(1+\frac{1}{n^2}\right)} = \frac{\frac{3}{n}-1}{n\left(1+\frac{1}{n^2}\right)} = \frac{\frac{3}{n}-1}{n+\frac{1}{n}}$$

$$\left.\begin{array}{l}\lim\limits_{n\to+\infty} \frac{3}{n}-1=-1 \\ \lim\limits_{n\to+\infty} n+\frac{1}{n}=+\infty\end{array}\right\}\text{donc, par quotient, } \boldsymbol{\lim\limits_{n\to+\infty} y_n = 0}$$